$$\log_{10}\sqrt{(x^2-1)}+\frac{1}{2}\log_{10}\left(\frac{x+1}{x-1}\right)$$

Solution

$$\log_{10}\sqrt{(x^2-1)}+\frac{1}{2}\log_{10}\left(\frac{x+1}{x-1}\right)$$

$$=\log_{10}(x^2-1)^{\frac{1}{2}}+\log_{10}\left(\frac{x+1}{x-1}\right)^{\frac{1}{2}}$$

$$=\log_{10}(x^2-1)^{\frac{1}{2}}+\log_{10}\frac{(x+1)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}}$$

$$=\log_{10}(x^2-1)^{\frac{1}{2}}\times\frac{(x+1)^{\frac{1}{2}}}{(x-1)^{\frac{1}{2}}}$$

$$=\log_{10}\left(\frac{(x^2-1)(x+1)}{x-1}\right)^{\frac{1}{2}}$$

$$=\log_{10}\left(\frac{(x-1)(x+1)(x+1)}{x-1}\right)^{\frac{1}{2}}$$

$$=\log_{10}((x+1)^2)^{\frac{1}{2}}$$

$$=\log_{10}(x+1)$$

$$\therefore \log_{10}\sqrt{(x^2-1)}+\frac{1}{2}\log_{10}\left(\frac{x+1}{x-1}\right)=\log_{10}(x+1)$$

Exercise 2*f*

1. Without using tables or calculator, find the values of the following:

(i) $\dfrac{\log_2 256}{\log_5 625}+\dfrac{\log_{10}81}{\log_{10}729}$

(ii) $\dfrac{\log_8 25+3\log_8 2}{2+4\log_8 5}$

(iii) $\log_{10}\left(\frac{35}{8}\right)+4\log_{10}2-\log_{10}7$

2. Express $\log_{10}\left(\frac{125}{128}\right)$ and $\log_{10}\left(\frac{625}{512}\right)$ in terms of $\log_{10}2$.

3. Express the following in terms of $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$.

(i) $\log_{10}\left(\frac{x^{\frac{1}{3}}y^4}{z^3}\right)$ (ii) $\log_{10}\left(\frac{100x^2}{y^3\sqrt{z}}\right)$

(iii) $\log_{10}\sqrt{\left(\frac{10x}{y^5z}\right)}$ (iv) $\log_{10}\frac{y\sqrt{x}}{\sqrt[3]{z}}$

4. Express the following as single logarithms.

(i) $3\log_{10}2+2\log_{10}3-2\log_{10}6$

(ii) $2\log_{10}x+3\log_{10}y-\log_{10}z$

(iii) $3\log_{10}x-\frac{1}{2}\log_{10}y+1$

(iv) $\log_7 98-\log_7 30+\log_7 15$

5. Given that $1+\log_{10}\left(\frac{4}{x^4}\right)^{-\frac{1}{2}}-2\log_{10}(x)=\log_{10}a$. Find the value of a.

2.2.4 The common logarithm

Common logarithms are logarithms used most often in numerical calculations. They are usually to the base of 10 and as such, the base is often omitted in written work. Thus $\log 2$ is usually taken to mean $\log_{10}2$.

Now

$$\log 2 = 0.3010$$

Thus, by index notation, we have:

$$2 = 10^{0.3010}$$

Examples

1. Given that $\log 2 = 0.301$ and $\log 3 = 0.477$, find the values of

(i) $\log 5$ (ii) $\log 6$

Solution

$\therefore x = -4$

Exercise 2h

1. Solve for x

   (i) $\log_x 0.001 = -3$

   (ii) $\log_{0.4} 0.0256 = x$

   (iii) $\log_{16} x = \frac{5}{4}$

   (iv) $\log_9(\sqrt[5]{27}/3) = x$

   (v) $\log_2(\log_2 x) = 2$

2. Solve the following equations

   (i) $\log(x-3) - \log(3-x) = 2$

   (ii) $\log(2x-1) - \log(x-3) = 1$

   (iii) $\log_a(x+2) - \log_a 3 = \log_a(x-4)$

Exercise 2*i*

1. Express the following in terms of the simplest possible surds.

(i) $\sqrt{72}$ (ii) $\sqrt{75}$

(iii) $\sqrt{50}$ (iv) $\sqrt{27}$

(v) $\sqrt{512}$ (vi) $\sqrt{1210}$

2. Express as a single square root

(i) $5\sqrt{5}$ (ii) $7\sqrt{2}$

(iii) $10\sqrt{7}$ (iv) $3\sqrt{17}$

(v) $3\sqrt{6}$ (vi) $3\sqrt{11}$

3. Simplify the following

(i) $\sqrt{50} + \sqrt{2} - 2\sqrt{18} + \sqrt{8}$

(ii) $\sqrt{512} + \sqrt{128} + \sqrt{32}$

(iii) $\sqrt{24} - 3\sqrt{6} - \sqrt{216} + \sqrt{294}$

(iv) $\sqrt{28} + \sqrt{175} - \sqrt{63}$

4. Given that $a\sqrt{2} + \sqrt{18} - \sqrt{50} = 3\sqrt{2}$

Find the value of a.

5. Express $\frac{\sqrt{2}}{2\sqrt{3}}$ in the form $\sqrt{\frac{a}{b}}$, where a and b are real numbers.

Exercise 2j

1. Simplify the following:

(i) $\frac{5\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ (ii) $\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{3}{\sqrt{3}-\sqrt{2}}$

(iii) $\frac{\sqrt{4}+\sqrt{3}}{\sqrt{4}-\sqrt{3}}$ (iv) $\frac{\sqrt{63}+\sqrt{28}}{\sqrt{175}-\sqrt{63}}$

2. Given that $\sqrt{3} = 1.732$ and $\sqrt{2} = 1.414$, evaluate $\frac{1}{\sqrt{3}-\sqrt{2}}$ to 3 significant figures.

3. Express $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$.

Hence evaluate $\frac{1+\sqrt{3}}{2+\sqrt{3}}$ correct to 3 significant figures if $\sqrt{3} = 1.732$.

4. Express $\frac{9}{\sqrt{5}-\sqrt{2}}$ in the form $a(\sqrt{b} + \sqrt{c})$, where a, b and c are integers.

5. Given that $\frac{5}{\sqrt{5}} + \sqrt{20} = a\sqrt{5}$, determine the value of **a**.

6. Given that $\frac{6}{3\sqrt{2}-2\sqrt{3}} = a\sqrt{2} + b\sqrt{3}$, find the values of a and b.

7. Express $\frac{2}{1+\sqrt{3}} - \frac{3}{3-2\sqrt{3}}$, without a surd in the denominator.

8. Express $\frac{1}{1-\sqrt{2}-\sqrt{3}}$, without a surd in the denominator.

Exercise 2k

Solve the following equations

1. $\sqrt{(3x-8)} - \sqrt{(x-2)} = 0$
2. $\sqrt{x-5} - \sqrt{x} = 2$
3. $\sqrt{(2x+3)} - 7 = 0$
4. $\sqrt{(3x+4)} = 4$
5. $\sqrt{10x} - 2\sqrt{5(x-5)} = 0$

Exercise 2n

[Use long division only]

1. Find the remainder when

(i) $12x^2 + 7x - 10$ is divided by $3x - 2$.

(ii) $20x^2 - 39x + 18$ is divided by $5x - 6$.

(iii) $x^4 - 6x^2 + 7x - 12$ is divided by $x + 3$.

(iv) $2x^3 - 5x^2 - 7x - 3$ is divided by $x + 1$.

(v) $2x^2 - 5x - 3$ is divided by $x - 3$.

(vi) $3x^2 - 2xy - 8y^2$ is divided by $x - 2y$.

(vii) $8y^3 - y^2 - y + 5$ is divided by $y^2 + y$.

(viii) $4x^3 - 5x^2 + 2x - 6$ is divided by $x^2 - 3x$.

(ix) $x^3 + 3x^2 + 3x + 1$ is divided by $x + 2$.

(x) $4x^3 - 6x + 5$ is divided by $2x - 1$.

2. Find the value of k that will make the divisor a factor of the dividend

(i) $\dfrac{x^3-5x^2-2x+k}{x-3}$

(ii) $\dfrac{x^3+3x^2-kx-12}{x+3}$

Exercise 2u

Solve the following equations

1. $x - 11\sqrt{x} + 30 = 0$

2. $x + 3\sqrt{x} - 10 = 0$

3. $2\sqrt{n} + 3 = n$

4. $\sqrt{(3x - 2)} = 3x - 2$

5. $\sqrt{(2t - 1)} + 2 = t$

6. $\sqrt{(x + 2)} - 1 = \sqrt{(x - 3)}$

Exercise 1B

1. Express each of the following as a single logarithm
   (i) $\log 2 + \log 3 + \log 5$
   (ii) $\log 6 + \log 3 - \log 9$
   (iii) $2\log a - \log b - \log c$
   (iv) $\log a + \frac{1}{2}\log b - 3\log c$
2. Solve the following, correct your answers to two decimal places
   (i) $2^{x-1} = 3^{x+1}$ (ii) $3^{x-2} = 2^{x+3}$
   (iii) $3^{2x-1} = 5^{x}$ (iv) $6^{1-x} = 2^{3x+1}$
3. Given that x and y are positive, solve the simultaneous equations $\log(xy) = 7$ and $\log\left(\frac{x}{y}\right) = 1$
4. Given that $\log(p - q + 1) = 0$ and $\log(pq) + 1 = 0$, show that $p = q = \frac{1}{\sqrt{10}}$
5. The variable x satisfies the equation $3^{x}.4^{2x+1} = 6^{x+2}$. By taking logarithms of both sides, show that $x = \frac{\log 9}{\log 8}$
6. Solve the pair of simultaneous equations
   $\log(x + y) = 0$
   $2\log x = \log(y - 1)$
7. Without using tables, show that
   $\frac{\log\sqrt{27} + \log\sqrt{8} - \log\sqrt{125}}{\log 6 - \log 5} = \frac{3}{2}$

Exercise 1C

1. Simplify each of the following

(i) $\sqrt{8}$ (ii) $3\sqrt{12}$

(iii) $\sqrt{125}$ (iv) 147

2. Express each of the following in the form $\frac{a\sqrt{c}}{b}$, where a,b and c are integers

(i) $\frac{2}{\sqrt{6}}$ (ii) $\frac{\sqrt{7}}{\sqrt{2}}$

(iii) $\frac{3\sqrt{5}}{2\sqrt{6}}$ (iv) $\frac{4\sqrt{45}}{5\sqrt{8}}$

3. Express each of the following in the form $\frac{a+b\sqrt{c}}{d}$, where a,b,c and d are integers

(i) $\frac{3}{6+\sqrt{3}}$ (ii) $\frac{3+\sqrt{2}}{5+\sqrt{2}}$

(iii) $\frac{6+\sqrt{5}}{2-\sqrt{5}}$ (iv) $\frac{3+\sqrt{24}}{2+\sqrt{6}}$

4. Express (i) $\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}+\sqrt{2}}$, (ii) $\frac{1+\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}$, with rational denominator

5. Simplify as possible $\frac{1}{3\sqrt{3}-3}+\frac{1}{\sqrt{5}+3}$

6. Express $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form $a+b\sqrt{3}$

7. Express $\frac{\sqrt{3}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ as equivalent fraction with a rational denominator

8. Simplify $\frac{(2+\sqrt{2})(3+\sqrt{5})(\sqrt{5}-2)}{(\sqrt{5}-1)(1+\sqrt{2})}$

Sample examination questions

1. (a) (i) Find $\log_9 27\sqrt{3}$ without using tables

(ii) Simplify $(\log_a b^2)\times(\log_b a^3)$.

(b) Express $\log_{25}(xy)$ in terms of $\log_5 x$ and $\log_5 y$.

Answers

(a) (i) 1.75 (ii) 6 (b) $\frac{\log_5 x+\log_5 y}{2}$

2. (a) Simplify $\frac{2\sqrt{6}-\sqrt{8}}{\sqrt{6}-\sqrt{5}}$

(b) Evaluate $\log_{0.4} 50$ (to 2 decimal places).

(c) Solve $2^{4(x-1)}=(4\times 8^x)^3$

Answers

(a) $7+\sqrt{30}$ (b) -4.27 (c) -2

3.(a) (i) Simplify $8^{\frac{2}{3}}$

(ii) If $4^x = 0.25$, find without using tables or calculate the value of x

(iii) if $\log_2 8^x = \frac{1}{\sqrt{3}}$, show that $x = \frac{\sqrt{3}}{9}$

(b) Evaluate to 3 significant figures $\frac{1}{1-\sqrt{3}}-\frac{1}{1+\sqrt{3}}$

Answers

(a) (i) 4 (ii) -1 (b) -1.73

4. Simplify:

(i) $4^{-\frac{1}{2}}\times 2^{n+3}\times 16^{-\frac{1}{2}}$

(ii) $\frac{\sqrt{3}-1}{2\sqrt{3}+1}$

(iii) $3\log a - 2\log b$

Answers

(i) 2 (ii) $\frac{7-3\sqrt{3}}{11}$ (iii) $\log a^3/b^2$

5. (a) If $y=3^2$ and $x=b^2$, simplify $y^{\frac{1}{2}}\times x^{\frac{1}{2}}$

(b) Given that $\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}=\frac{y-2}{3^{-\frac{1}{2}}}$, find without using tables or calculate, the value of y.

(c) without using tables or calculator, use $\log_{10} 2 = 0.301$ and $\log_{10} 5 = 0.699$ to find the value of: (i) $\log_{10} 2.5$ (ii) $\log_{10} 6.25$

Answers

(a) ab (b) $\sqrt{6}$ (c) (i) 0.398 (ii) 0.796

Sample examination questions

1.(a) Express $2x^2 + 11x + 6$ in the form $p(x+q)^2 + r$ and hence deduce its minimum value, where p, q and r are constants.

(b) If α and β are the roots of $ax^2 + bx + c = 0$, express $(\alpha - 2\beta)(\beta - 2\alpha)$ in terms of a, b, and c. Hence deduce the condition for one root to be twice the other.

Answers

(a) $2\left(x + \frac{11}{4}\right)^2 - \frac{73}{8}$, $-\frac{73}{8}$

(b) $\frac{9ac - 2b^2}{a^2}$, $2b^2 = 9ac$

2. Solve $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$

Answer: 0, -1

3. Show that $2x^2 - 36x + 175$ may be written in the form $a(x-b)^2 + c$, where the values of a, b, and c are to be found. Find the minimum value of $2x^2 - 36x + 175$ and state the value of x for which this minimum occurs

Answer: $(x-2)^2 + 13$. Min value = 13 at $x = 2$

4. Given that for all values of x,

$3x^2 + 12x + 5 \equiv p(x+q)^2 + r$

(a) Find the values of p, q and r

(b) Find the minimum value of $3x^2 + 12x + 5$

(c) Solve the equation $3x^2 + 12x + 5 = 0$

Answer:

(a) $p = 3, q = 2, r = -7$ (b) -7 (c) -3.5, -0.5

5. The quadratic equation $x^2 + 6x + 1 = k(x^2 + 1)$ has real roots. Find the possible values of the constant k

Answer: -2 or 4

2. Factorise the following quadratic trinomials.

(i) $x^2 + 5x + 6$

(ii) $x^2 + x - 20$

(iii) $y^2 + 15y + 36$

(iv) $5t^2 + 28t + 32$

(v) $2x^4 + x^2 - 6$

(vi) $x^4 - 8x^2 + 15$

(vii) $4x^2 + 5x - 9$

(viii) $15c^2 - 34c + 16$

(ix) $2p^2 - 27p + 36$

(x) $28a^2 + 13a - 6$

(xi) $2m^2 - 11m + 15$

(xii) $3x^2 - 24x - 27$

(xiii) $12x^2 + 36x + 27$

(xiv) $16x^2 - 80x + 100$

3. Solve the following quadratic trinomials by factorization.

(i) $x^2 - 7x + 12 = 0$

(ii) $x^2 + 9x + 20 = 0$

(iii) $y^2 - 3y = -2$

(iv) $6t^2 + 23t - 4 = 0$

(v) $24x^2 + 23x - 12 = 0$

(vi) $2x^2 - 13x + 6 = 0$

(vii) $3x^2 + 5x - 12 = 0$

(viii) $15c^2 + 19c - 10 = 0$

(ix) $10x^2 + 33x - 7 = 0$

(x) $4p^2 + 9 = 12p$